
ECONOMIC EXCHANGE AND SOCIAL ORGANIZATION

The Edgeworthian foundations of
general equilibrium theory

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VOLUME 12

Editor: S. H. Tijs (University of Tilburg); *Editorial Board:* E.E.C. van Damme (Tilburg), H. Keiding (Copenhagen), J.-F. Mertens (Louvain-la-Neuve), H. Moulin (Durham), S. Muto (Tohoku University), T. Parthasarathy (New Delhi), B. Peleg (Jerusalem), H. Peters (Maastricht), T. E. S. Raghavan (Chicago), J. Rosenmüller (Bielefeld), A. Roth (Pittsburgh), D. Schmeidler (Tel-Aviv), R. Selten (Bonn), W. Thomson (Rochester, NY).

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The Edgeworthian foundations of
general equilibrium theory

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KLUWER ACADEMIC PUBLISHERS
Boston/London/Dordrecht

Distributors for North America:

Kluwer Academic Publishers
101 Philip Drive
Assinippi Park
Norwell, Massachusetts 02061 USA

Distributors for all other countries:

Kluwer Academic Publishers Group
Distribution Centre
Post Office Box 322
3300 AH Dordrecht, THE NETHERLANDS

Library of Congress Cataloging-in-Publication Data

Gilles, Robert P.

Economic exchange and social organization : the Edgeworthian foundations of general equilibrium theory / Robert P. Gilles.

p. cm. -- (Theory and decision library: Series C, Game theory, mathematical programming, and operations research ; v. 12)

Includes bibliographical references and index.

ISBN-13: 978-1-4612-8549-6

e-ISBN-13: 978-1-4613-1285-7

DOI: 10.1007/978-1-4613-1285-7

1. Equilibrium (Economics) 2. Institutional economics. 3. Social structure. 4. Exchange. 5. Barter. 6. Edgeworth, Francis Ysidro, 1845-1926. I. Title. II. Series

HB145.G55 1996

330'.01'5195--dc20

96-32595

CIP

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Softcover reprint of the hardcover 1st edition 1996

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Printed on acid-free paper.

Each human being depends for survival on the immediate and broader surrounding society. Human beings are not the independent windowless Leibnitzian monads sometimes conjured up by libertarian theory. Society is not imposed on humans; rather, it provides the matrix in which we survive and mature and act on the environment. Families and the rest of society provide nutrition, shelter, and safety during childhood and youth, and then the knowledge and skills for adult performance. Moreover, society can react to a person's activities at every stage of life, either facilitating them or severely impeding them. Society has enormous powers, enduring through a person's lifetime, to enhance or reduce evolutionary fitness.

Citation from Herbert A. Simon (1991), "Markets and Organizations", Journal of Economic Perspectives, 5, 25-44.

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PREFACE

This book aims to develop an institutional approach to general economic equilibrium. Thus far, institutional economics has essentially been confined to purely verbal discourse. Here I argue the case that general equilibrium theory forms a well rounded basis for the development of an *institutional economic theory*. The fundamental economic trade mechanism underlying this refocusing is that of the Edgeworthian barter mechanism modelled through the equilibrium notion of the *core* of an economy. There is an extensive literature that links the core with the Walrasian price mechanism, which is explored in this book. Next I develop an alternative model of explicitly nonsovereign trade in the setting of an institutionally structured economy. In this book the core and several of its extensions are considered to be descriptions of the equilibrium allocations resulting from institutionalized barter processes, thereby providing a basis of an institutionally based economic theory.

Traditionally finite economies have been assessed as the most natural representations of real life economies, in particular of market economies. Many fundamental insights have been developed. In the first half of the book I summarize the most influential and important results in the literature on finite economies regarding the relationship of the Walrasian model of a perfectly competitive market system and the Edgeworthian theory of individually based, pure barter processes. I use the axiomatic method as the main methodological framework according to which I construct my models.

Next I turn to the development of the Walrasian model and the traditional Edgeworthian concept of the core of an economy. Existence of Walrasian equilibria and core allocations are discussed. To achieve a better understanding of Edgeworth's barter mechanism I summarize some of the literature on trade and negotiation processes underlying this notion: a discussion of Graham and Weintraub's barter process and Green's negotiation process is given. The relationship of the Walrasian and Edgeworthian models is explored further by investigating equity properties of core allocations, the introduction of replica economies and the Debreu-Scarf convergence theorem, which states that topologically the core of a replicated economy converges to the set of Walrasian

equilibrium allocations. A second type of approximation result investigates whether core allocations approximately have the same properties as Walrasian equilibrium allocations. Both methods indicate that largeness of the economy is crucial in the establishment of perfect competition among the agents, thereby confirming Edgeworth's original conjecture.

In the second half of this book I turn to the description of institutionally based trade processes. I closely follow the conventional model of a large economy in the design of a two-level institutional model of social interaction. At the first, lower level of interaction, primitive institutional coalitions of neighboring economic agents are introduced. At the second, higher level of interaction these primitive coalitions merge into trade coalitions. If the two levels coincide, we arrive at Aumann's model of a large economy. Within this model Walrasian equilibrium allocations can be determined and compared with the allocations achieved through Edgeworthian barter processes with imperfect coalition formation described in the two-level institutional model of economic interaction. The traditional question, under which conditions equivalence between these two organizations of economic exchange is achieved, is chosen as the focal point of the investigations.

Different assumptions on the ability of primitive coalitions to redistribute their resources among their members lead to different variations of the core. I propose two generalizations of the traditional core in this framework. The first core concept, called the *semi core*, describes barter in an institutional environment where free re-distribution of resources among members of a trade coalition is allowed. Hence, only institutional constraints on coalition formation are implemented. The second variation, called the *contract core*, additionally requires equal treatment of the members of participating institutions within a trade coalition. For both core concepts conditions for equivalence with the Walrasian equilibrium economy under the hypothesis of a perfect price mechanism are identified. These results lead, however, to the same conclusion: if the institutional structure of the economy is dense and flexible, then, in general, the institutionally based Edgeworthian barter processes result in Walrasian equilibrium allocations. As a corollary we derive Aumann's core equivalence theorem.

These insights necessitate a further investigation of the conditions under which core-Walras equivalence holds. From this investigation, it is concluded that this equivalence is essentially an anomaly: the two-level institutional model of economic interaction indicates the impossibility of the simultaneous satisfaction of the density and flexibility requirements regarding the institutional trade infrastructure. The institutional nature of the trade infrastructure is counter-intuitive to any form of flexibility. This should not be viewed as a discouraging

insight, but rather valued as a stimulus for further research into the delicate balance of institutionalized economic interaction on the one hand and necessary flexibility to achieve efficient outcomes on the other.

Acknowledgements

This work came largely forth from my doctoral dissertation as defended at Tilburg University, Tilburg, the Netherlands, in 1990. Over the past years this material has been transformed into an extended text on general economic equilibrium theory. Previous versions served as lecture notes for courses on general equilibrium theory at the Department of Economics at Tilburg University as well as at the Virginia Polytechnic Institute and State University.

Through the development of this book many persons have contributed to its content and form. First and foremost I thank my advisor Pieter Ruys for the many discussions that stood at the origin of many fruitful ideas and theories presented in this volume. Many of these ideas would not have matured without his support and enthusiasm. Second, I am very grateful to my friend and colleague Dimitrios Diamantaras, who carefully read the sometimes irregular manuscript and pointed out numerous mistakes. His input gave my manuscript its current form. Furthermore, I thank Hans Haller for his thorough investigation of the material covered by this book and his contributions at many points in the text. Also I would like to thank Sudipta Sarangi, Dolf Talman, and Willy Spanjers for their advise and extensive comments on previous drafts of the text. Finally, I am grateful to all students who over time contributed many useful alterations and suggestions to the different versions of the manuscript I used in their classes. In particular I would like to thank Anthony Pavlopoulos and René van den Brink. Their feedback has been extremely valuable in the development of the theory presented.

Robert P. Gilles

Blacksburg, June 4, 1996

SCARCITY AND GAINS FROM TRADE

In this book I intend to give an overview of the insights as developed in the theory of general economic equilibrium. Besides presenting the main results in the literature, I develop several new insights. As the name suggests, *general equilibrium theory* addresses the modelling of a global economy, i.e., it describes the totality of interactions among all economic decision making institutions and persons in a society. These decision makers include individuals, households, firms, labor unions, non-profit organizations, governmental agencies, political parties as well as lobby groups such as the ACLU and the NRA. The ultimate goal of general equilibrium theory is to extend our understanding of global human economic interaction and the consequences of changes in that interaction.

Although the agenda for general equilibrium theory seems very ambitious, economists have developed some important insights into the nature of the outcomes of such global economic interaction. Unfortunately, due to the enormous complexity of our society, these achievements only partially reveal the nature of the consequences of global economic interaction. It is my intention to show that a further development of general equilibrium theory is necessary to extend these insights and to give direction to this development. This contrasts the general sentiment among contemporary game theorists, who defend the point of view that insights from *partial equilibrium* models rather than general equilibrium models are more valuable in the better understanding of our economic problems. I take the position that both partial and general equilibrium models should be developed further to enhance our insights.

In this introductory chapter I limit myself to a sketch of the principles of modelling general equilibrium. I do this by using some examples to illustrate

the problems that have to be addressed in the design of general equilibrium models. Second, I turn to the different schools of thought that have emerged during the development of this general theory of value.¹ Before turning to the discussion of the general principles of economic interaction I emphasize that this approach has been set up in the strict fashion of the axiomatic method: each economic concept is represented by a mathematical notion or theory and the theorems as derived within this mathematical representation are the logical conclusions of the economic modelling process as pursued by the theorist. In the following chapter I will discuss the axiomatic construction of microeconomic theories in more detail.

1.1 GAINS FROM TRADE

Since Robbins (1932) formulated the notion of *economic scarcity* as the foundation of the market- or price mechanism, it has generally been accepted that the fundamental principle on which people pursue economic interests is that of the accumulation of gains from trade. Scarcity refers to the principle that economic resources are fundamentally limited, while there are unlimited ways to use these resources. As a consequence economic subjects have the potential to increase their wealth by exchanging their scarce resources, but can do so in numerous ways. Thus, there is a fundamental *choice problem*. The increase in wealth of the economic subjects is usually referred to as *gains from trade* realized in the process of economic exchange. To illustrate the traditional treatment of the issue of economic scarcity and the accumulation of gains from trade I discuss a simple economic situation.

Example 1.1.1 Consider two economic subjects, say Anne (a) and Ben (b). There are two scarce resources, namely a composite consumption commodity — also known as *money* — and a composite service commodity — usually indicated as *labor*. Quantities of these commodities are indicated by two dimensional real valued vectors $(x_1, x_2) \in \mathbb{R}_+^2$. Anne owns one unit of money, while Ben has one unit of labor. In the sequel we denote these initial endowments by $w(a) = (1, 0)$ and $w(b) = (0, 1)$. The total initial resources in this situation are thus given by $w(a) + w(b) = (1, 1)$.

Both Anne and Ben have the same Cobb-Douglas preferences given by the util-

¹The *theory of value* is another name for general equilibrium theory. It refers to the classical nineteenth century program into the investigation of the determinants of economic value. Market theory introduced by Walras is only one of the options explored in this context. The famous labor theories of value of Marx and Ricardo also come to mind.